

Exercise 52

Find an equation of the tangent line to the curve at the given point.

$$y = \sqrt{1 + x^3}, \quad (2, 3)$$

Solution

A point on the tangent line is known, so all that's needed is its slope. Take a derivative of the given function

$$y' = \frac{d}{dx} \sqrt{1 + x^3} = \frac{1}{2}(1 + x^3)^{-1/2} \cdot \frac{d}{dx}(1 + x^3) = \frac{1}{2}(1 + x^3)^{-1/2} \cdot (3x^2) = \frac{3x^2}{2\sqrt{1 + x^3}}$$

and evaluate it at $x = 2$.

$$y'(2) = \frac{3(2)^2}{2\sqrt{1 + (2)^3}} = 2$$

Therefore, the equation of the tangent line to $y = \sqrt{1 + x^3}$ at $(2, 3)$ is

$$y - 3 = 2(x - 2).$$

Below is a graph showing the function and the tangent line.

